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1991 J. Phys.: Condens. Matter 3 1961

(http://iopscience.iop.org/0953-8984/3/12/028)

J. Phys.: Condens. Matter 3 (1991) 1961–1965. Printed in the UK

LETTER TO THE EDITOR

Transmission coefficients and Hall resistance in a small cross-shaped semiconductor junction

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Received 19 December 1990

Abstract. A method is described that measures both the transmission coefficients and the Hall resistance for a narrow cross. Büttiker's formula relating these quantities has been verified. The amount of collimation is estimated from the magnetoresistance and shown to be small, with ≃5% more electrons travelling straight on at the cross in zero magnetic field compared with a magnetic field of 0.1 T. This illustrates that the reason for the quenching of the Hall effect here is a scrambling mechanism rather than the collimation mechanism.

The disappearance of the Hall effect at low magnetic fields was first observed by Roukes *et al* (1987) in Hall bar structures with widths of order 0.1 μm. In this regime the electrons travel ballistically through the whole device region—see, for example, Wharam *et al* (1988a, b) and von Wees *et al* (1988). Current explanations of this result focus on models in which the probes are strongly coupled to the channel and cause the quenching. These approaches use the Büttiker (1986) formula

$$R_{mn,kl} = (h/e^2)(T_{km}T_{ln} - T_{kn}T_{lm})/D \qquad (1)$$

to find the four-terminal resistance of the cross. The transmission coefficient T_{km} is the transmission probability from k to m, $R_{mn,kl}$ is a four-terminal measurement with mn the current probes and kl the voltage probes, and D is a subdeterminant of order 3 of the matrix $1 - \mathbf{T}$. Baranger and Stone (1989) use this equation to predict the Hall resistance. Their model includes a soft-walled potential in the vicinity of the cross, coupled to hard walls (of infinite potential) as the leads. In this model, they separate two causes of quenching, both of which are not present for a perfect cross. One cause is that the probabilities of transmission into the left-hand and right-hand leads become very similar; the other is that the electrons are collimated, and all end up going straight ahead at the cross. Baranger and Stone emphasize the collimation mechanism, in which both the transmission coefficients into the side contacts are small, and demonstrate that in their model it is significantly more important than the reduction in asymmetry of the transmission coefficients.

Ford *et al* (1989) investigated the quenching using a number of differently shaped central regions. They were able to change the measured Hall effect by shaping the sample to bounce the electron into the left-hand probe when the magnetic field bent the electron trajectory to the right. In this way they were able to change the sign of the Hall

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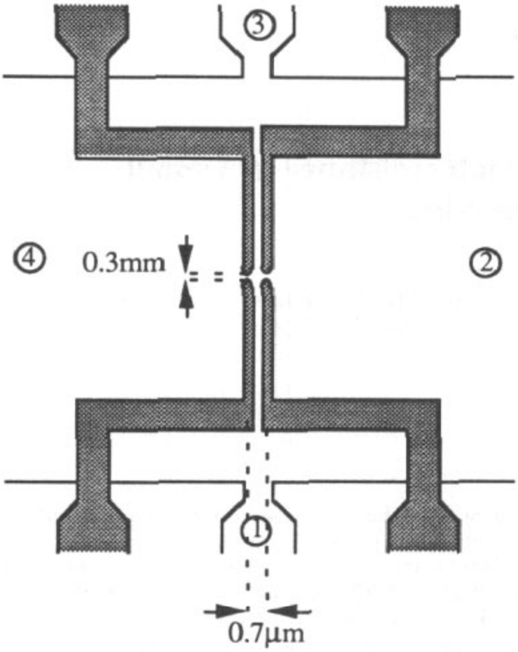

Figure 1. A schematic diagram of the double-split-gate device used, with the gate shaded.

effect at certain fields. This provides evidence in favour of a significant randomization of the direction of the electron in the cross. Beenakker and van Houten (1989) similarly give more weight to the scrambling mechanism, in which electrons are randomized by the cross at very low magnetic fields and hence emerge from all of the contacts in roughly equal numbers.

Main *et al* (1990) have calculated the transmission coefficients from measurements of the resistance and the Hall resistance using (1), and have shown that the transmission coefficient into the direction favoured by the magnetic field rises to 1 when the cyclotron radius is much smaller than the effective sample size.

In this letter, we take a different approach to previous experimental studies. Instead of assuming the Büttiker formula we measure both the transmission coefficient and the Hall effect. This allows the relationship between these two quantities to be experimentally verified. Furthermore, the direct measurement of the transmission reveals the collimation effect to be small at low magnetic fields, showing that here the quenching of the Hall effect is due to scrambling.

The structure used for this work is described in Wharam *et al* (1988a, b) and shown schematically in figure 1. The starting material used is a high-mobility piece of modulation-doped GaAs–AlGaAs heterostructure (wafer C187). Before processing, the mobility was $1.70 \times 10^6\,\mathrm{m^2\,V^{-1}\,s^{-1}}$ with a photoinduced carrier concentration of $3.5 \times 10^{15}\,\mathrm{m^{-2}}$. Two split gates are fabricated close together on a piece of the high-mobility heterostructure. When a negative voltage is applied to the gates the electrons in the heterostructure are depleted underneath, leaving the active region of the device in the form of a cross.

Most of the results presented in this paper were taken with a gate voltage of −0.2 V applied to each gate. This is sufficient to define the sample, but does not excessively increase the resistance of the long channels, such as contact 2 in figure 1. At this gate

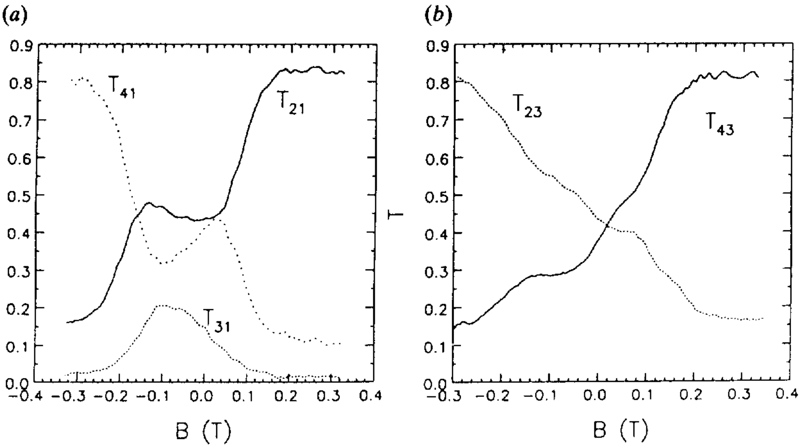

Figure 2. Transmission coefficients. In (*a*), T_{21} is the full curve, T_{31} the densely dotted curve and T_{41} is the sparsely dotted curve. In (*b*) T_{43} is the full curve and T_{23} the dotted curve. Note the asymmetry, i.e. T_{21} and T_{43} are not identical. The high-field regions demonstrate the final plateau.

voltage the two-dimensional carrier concentration is $1.72 \times 10^{15}\ \mathrm{m}^{-2}$ measured from the slope and plateaux positions of the high-field Hall effect.

If a known current is passed into the device along one of the leads the transmission coefficient can be obtained by simply measuring the current emerging from each of the other three grounded contacts. For example, consider the case where the current enters through contact 1. The split gates provide an intrinsic resistance of $(h/e^2)N$ (where N is the number of sub-bands in the contact) which is much higher than the resistance to ground. Therefore, virtually all of the electrons that are transmitted right at the cross flow to ground via contact 2. This means that the fraction of the injected current measured in contacts 2 and 4 is a direct measure of the transmission probability to the right and to the left at the cross.

In order to take measurements using very small currents, an AC lock-in technique is used. A constant current is injected by using an oscillator of constant voltage in series with a $10^8\ \Omega$ resistor. Two of the other three contacts are grounded directly, whilst the third is connected to the input of a Brookdeal 5002 preamplifier, which acts as a virtual ground. In order to prevent the DC voltage present on the preamplifier input interfering with the experiment, a large blocking capacitor was used between the preamplifier and the sample. The preamplifier output was passed into a lock-in amplifier to detect the signal. The circuit was calibrated by feeding the current directly from the resistor into the blocking capacitor and preamplifier; in this way it could be shown that the blocking capacitor made no significant difference to the AC signal at the frequency used ($\simeq$70 Hz). It was also checked that the sample was in the linear regime by reducing the current by factors of 2 and 10 and observing a corresponding reduction in output.

A complication occurs because the transmission into the long wire contact is not ballistic, and some electrons will be reflected back to the junction as they travel along the narrow channel. To minimize this effect the channels are not strongly pinched off. The amount of this reflection can be estimated, by sending current in through contact 2 and measuring the output at contact 3, with a magnetic field applied that is large enough to guide most of the electrons down contact 3. The measured transmission coefficient

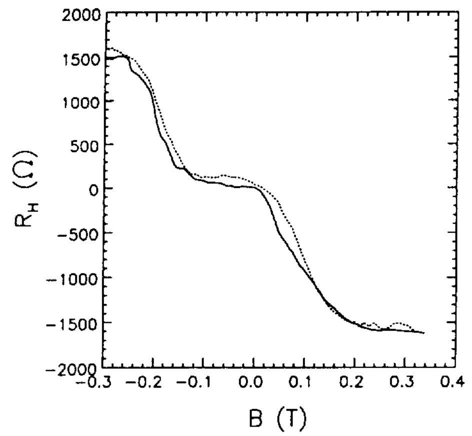

Figure 3. The calculated (dotted curve) and measured (full curve) Hall effects.

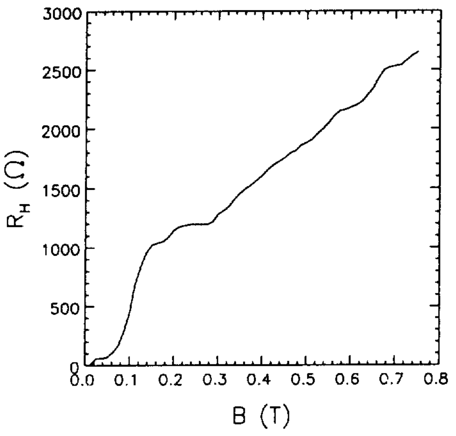

Figure 4. The moderate-field Hall effect showing the quenched region near the origin and the final plateau at about 0.2 T.

into contact 3 is about 0.5. Using this information, a simple correction can be made to the experimentally measured transmission coefficients to give a measure of the real transmission coefficients at the junction. Although this is about 50%, the correction is only appreciable for fields near zero, where a significant current flows straight ahead. The correction results in virtually no change to the Hall resistance predicted from Büttiker's formula (1).

Figure 2 shows some of the transmission coefficients measured for a gate voltage of −0.2 V. There is a slight offset in the magnetic field scale, due to a small magnetic field remaining in the large superconducting magnet in the cryostat. The curves shown are asymmetric, probably due to device imperfections which are most probably impurities in the region of the centre of the device. The measured transmission coefficients are only very weakly dependent on temperature. Curves of the resistance against the gate voltage demonstrate that at this voltage the two-dimensional electron gas is depleted under the gate, but that none of the four probes leading to the cross are pinched off.

The Hall effect is observed to be quenched, as shown in figure 3, for measurements in the same conditions as for the curves in figure 2. It should be noted that the Hall effect still shows a good flat region, in spite of the asymmetries of the transmission coefficients which give rise to the resistance.

It is possible to check Büttiker's formula for the resistance, equation (1), because the experiment gives an independent measure of the Hall resistance and the transmission coefficients that cause the resistance.

The first difficulty to be solved in using the formula in this case is that the measured transmission coefficients vary between 0 for no transmission and 1 for complete. However, equation (1) is only for a single sub-band, and to use it directly for a multiple-sub-band system the transmission coefficients must vary between 0 and N, where N is the number of sub-bands in the contact feeding in the current (Büttiker 1988). The obvious way to find the number of sub-bands in a ballistic split gate sample is to use the Landauer formula for the resistance and assume $T = 1$. The resistance can be used to obtain an estimate of $N \simeq 9$ spin-degenerate sub-bands. This estimate must be treated with caution because at this moderate gate voltage the quantized plateaux, first observed by Wharam *et al* (1988a, b) and van Wees *et al* (1988), are not well resolved. If T is less than 1, then N will be above 9.

The alternative approach is to find the number of sub-bands from the width which can be estimated from the position of the final plateau following Beenakker and van Houten (1989). From the observed field B for the start of the final plateau, (0.20 ± 0.03) T, the width is (0.4 ± 0.1) μm. This agrees with the lithographic width of (0.4 ± 0.1) μm when the device is just defined. Using the parabolic model of Berggren *et al* (1988) this width corresponds to 9 sub-bands. However a simple infinite square-well potential will push this estimate up to 12. The actual value probably lies between these two extreme models.

Therefore both the transmission coefficient and the width estimates suggest an N of 10 or 11. Taking $N \simeq 11$ predicts a Hall effect shown as the dotted curve in figure 3. The predicted and measured Hall effects agree well, which confirms Büttiker's formula.

The results shown in figures 2 and 3 show that a small low-field Hall resistance can be obtained even with transmission into the side contacts. The corrected transmission coefficients at zero magnetic field are all roughly the same. This demonstrates that the collimation mechanism is not the key effect and that the reason for the quench is the scrambling mechanism. From a classical point of view the electrons bounce around in the interior region of the cross and gain sufficient momentum perpendicular to their injection direction to leave the cross down the probes at the side. An assumption of a smooth change in the eigenstate as the electron leaves the long channel regions and enters the cross must break down, either because of scattering off the walls, or because the potential changes too quickly. It should be emphasized that this does not mean that there is no collimation effect, just that it is not the dominant reason for quenching.

In summary, we have aimed to verify experimentally Büttiker's formula relating the transmission coefficient to the four-terminal resistance. This agreement is shown in figure 4.

We would like to thank the European Community under the auspices of ESPRIT Basic Research action 3043, the Science and Engineering Research Council and the European Research Office of the US Army for supporting this work.

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